Determination of the bulk elastic moduli of various concrete by resonance frequency analysis of slabs submitted to degradations

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Abstract

The study presented here aims at evaluating the bulk elastic Young modulus of six different concrete mixes as a function of the water content and degradations due to carbonation or chloride ingress. The frequency analysis of ultrasonic waves in concrete after the impact of a steel ball (impact echo method) is commonly used to measure the thickness of large slabs, to detect voids in concrete structural elements considered as infinite. In the research project, this method was employed on reduced sized slabs (0.5x0.25x0.12 m^3). As a consequence, it was necessary to identify the frequencies corresponding to resonance modes or pseudo-stationary modes. This modal analysis was validated by several simplified models (for thin or semi-thick slabs or beam) and used to calculate the dynamic Young modulus E_{dyn} and the Poisson ratio. This last parameter is varying from 0.17 to 0.24, classical values for concrete. The dispersion of the Poisson ratio is too important and the values can not be compared to destructive nor non destructive test results. Otherwise once inverted for all concrete mixes, the E_{dyn}-modulus is compared to static Young modulus E_{stat} measured by destructive testing.

Résumé

L'étude présentée ici vise à évaluer le module de déformation élastique de neuf bétons différents en fonction de leur teneur en eau et des dégradations dues à la carbonatation et à la pénétration des chlorures. L'analyse fréquentielle d'ondes mécaniques se propageant dans le béton suite à l'impact d'une bille d'acier (méthode impact-écho) est utilisée habituellement pour mesurer l'épaisseur de dalles ou pour détecter des défauts dans des éléments de structure considérés comme infinis. Dans le projet, cette méthode a été employée sur des dalles de taille réduite (0.5x0.25x0.12 m^3). Par conséquent, il a été nécessaire d'identifier les fréquences correspondant à des modes de résonance ou à des modes pseudo-stationnaires. Cette analyse modale a été validée par deux modèles simplifiés (pour des dalles minces et des poutres) et utilisé pour calculer le module d'Young dynamique E_{dyn} et le coefficient de Poisson. Ce dernier paramètre varie de 0.17 à 0.24, valeurs classiques pour le béton. La dispersion des valeurs obtenues pour le coefficient de Poisson est importante et de plus celles-ci ne peuvent pas être comparées à des coefficients similaires obtenus par méthodes destructives ou non-destructives. Par ailleurs, une fois inversé pour toutes les formulations de béton, le module dynamique E_{dyn} est comparé au module d'Young E_{stat} mesuré par essai destructif.

Keywords

Impact echo, porosity, water content, carbonation, concrete.

1 Introduction

As the part of the French national research project SENSO, nine different concrete mixes were submitted to Non Destructive (ND) tests to evaluate their durability indicators (porosity for instance), water content, degradation monitoring parameters and their mechanical characteristics [1]. In this paper the results obtained with the impact-echo method applied on small size concrete slabs are presented. In order to evaluate the Young modulus E_{dyn} and the Poisson ratio ν, it is necessary to identify the resonance frequencies of these slabs. Simplified models (thin slab or beam theories) are used to obtain these frequencies as functions of E_{dyn}, ν, the density ρ and the slab dimensions. The inversion to calculate E_{dyn} and ν is then performed on all the studied concretes and compared to destructive results (static deformation modulus E_{stat} and porosity ϕ) and other ND ultrasonic surface wave results.

2 Impact echo method

2.1 Principle of the method

The principle of the impact-echo method consists in a frequential analysis of mechanical waves propagating in a concrete structure following a shock of a steel ball [2]. The shock and the hand-held transducer (that records a voltage signal proportional to the surface displacement) are located at short distance one of the other in the center of the upper slab surface. The FFT of the temporal voltage signal is then used to measure slab thickness or to localize defaults [2,3]. For an infinite size slab, the frequency (characteristic of the thickness) corresponding to the frequency at which the group velocity of the first symmetric Lamb mode (S1 mode) is equal to zero [4]. In this study, the tests being carried out on slabs of reduced dimensions, this particular pseudo-stationary frequency must be found among the various resonance frequencies of the slab (see Figure 1).

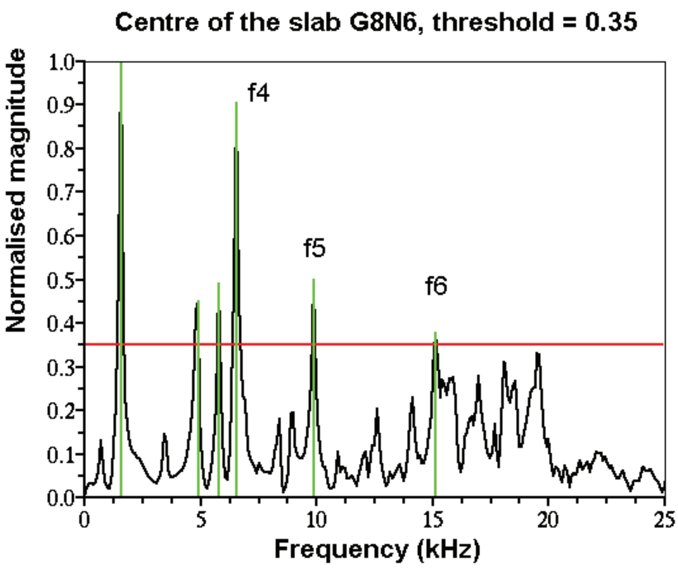

Figure 1. *Impact-echo typical spectrum for the concrete slab G8N6 in a saturated state*

1.2 Resonance frequencies of the concrete slabs: thin slab theory

The analysis of specific frequencies of the most important amplitudes makes it possible to calculate the dynamic elastic module E_{dyn} and the Poisson ratio ν, by using the densities measured on the slab (Table 1). Here we focused on the frequencies labeled *f4, f5* and *f6* in Figure 1. For instance, the recorded frequencies for the concrete G8 in saturated conditions are *f4=6357±49Hz, f5=9663±49Hz* and *f6=14934±49Hz.*

For thin plates, the Kirchhoff model is generally used as direct model. This model neglects the shear slipping and assumes that cross-sections are flat and perpendicular to the reference plane (before deformation). The model gives good results for the lowest first bending modes and for thin plate where length (*L*) over thickness (*e*) ratio is larger than 20. In the presented case *L/e=4.16* but this model would give a first approximate. Moreover, such simple model has the main advantage to provide an explicit formulation of modal displacements and frequencies that can be used for characterization [5]. In the case of free boundary conditions, the bending modes are defined by the following frequencies:

$$f_{n,m} = \frac{\pi}{2}\left[\left(\frac{n}{l}\right)^2 + \left(\frac{m}{L}\right)^2\right]\sqrt{\frac{E_{dyn}e^2}{12\rho\left(1-\nu^2\right)}} \tag{1}$$

In Figure 1, the frequency *f4* is identified to be the $f_{1,2}$ modal frequency. Typical values are given in Table 1.

Table 1. *Resonance frequencies of a saturated concrete G8 slab (ρ=2404kg/m^3, E_{dyn}=30.7GPa, n=0.2, e=0.12m), thin slab Kirchhoff's theory*

l=0.25m , L=0.50m	m=1	m=2	m=3
n=1	3969	**f4 = 6350**	10319
n=2	13494	15875	19843

Moreover, other resonance frequencies are measured during the dynamical analysis. Among these various frequencies, we can observe bulk shear waves *f5*

$$f5 = f_s = \frac{1}{2e}\sqrt{\frac{E_{dyn}}{2\rho(1+\nu)}} \tag{2}$$

The last mode that we can observe in this frequency range is the first symmetric Lamb mode (for group velocity equal to zero) expressing dilatation/compression of the section. Its frequency has the following expressions:

$$f6 = f_{S1} = \frac{\Omega_{S1}}{2e}\sqrt{\frac{E_{dyn}}{2\rho(1+\nu)}} \tag{3}$$

In Eq.3 the factor Ω_{S1} is only function of the Poisson ratio [4]. For the concrete G8, *f5=9610 Hz* and *f6=14957 Hz* are obtained.

2.2 Resonance frequencies of the concrete slabs: beam theory

In order to extend the validity range of the model, the first idea is to use the Mindlin theory of semi-thick slabs (valid for $4 \le L/e \le 20$) [5]. This model takes into account the transversal shear and rotary inertia and gives efficient prediction at higher frequency. In the case of free-free boundary conditions the first bending mode are invariant along one direction. The bending behavior of the plate is almost equivalent to that of a beam. It is then possible to use a Timoshenko beam model to evaluate the first bending frequencies. This model used the same hypotheses than Mindlin model: predictions are more accurate for high frequencies.

The natural frequencies of the Timoshenko [6] beam model are the roots of the following dispersion relation

$$\omega^4 - \frac{\kappa G}{\rho}\left(\frac{A}{I} + \left(\frac{n\pi}{L}\right)^2\left(1 + \frac{E}{\kappa^2 G}\right)\right)\omega^2 + \frac{\kappa^2 EG}{\rho^2}\left(\frac{n\pi}{L}\right)^4 = 0 \tag{4}$$

where I is the moment of inertia, A is the area of the cross section, G is the shear modulus, and κ is a shape factor. Note that the problem must be addressed for each direction in order to evaluate bending along the each large faces of the slab. In particular, A and I must be evaluated separately for each directions.

The dynamical investigation in terms of beam model allows to increase the accuracy of the analytical evaluation of the first bending modes given by Kirchhoff model.

3 Analysis of the experimental results

3.1 Characteristics of the concrete mixes and the concrete slabs

In project SENSO [1], 9 concrete mixes were manufactured, characterized mechanically and tested by various nondestructive methods. In this paper, the results obtained for only 6 concretes (named G1, G2, G3, G3a, G7 and G8) mixed with the same cement, the same aggregates but with different water to cement ratios in order to have porosities ranging from 12 to 18%. We can note that the concrete G1 comprises silica fume. Destructive tests are performed on cylindrical specimens to characterize the concretes (see Table 1).

Nondestructive measurements were carried out on 9 slabs of 50x25x12 cm^3 for each concrete at various degrees of saturation (roughly 0%, 40%, 60%, 80% and 100%). The dry state is reached by drying in an oven at 80°C for a minimum of 2 months and the saturated state by immersion in water for 3 months approximately. Then, part of the slabs is saturated by a salt solution (30g/L or 120g/L) in a similar way (concretes G1, G3, G8) whereas another part (concretes G3, G3a, G7 and G8) is carbonated at different depths according an accelerated process (50% of carbon dioxide at a relative humidity of 50%).

The impact-echo measurements are performed three times at the center of upper face of each slab. The temporal signals are averaged before the spectrum calculation.

Table 1. *Concrete characteristics in saturated conditions*

		G1	**G2**	**G3**	**G3a**	**G7**	**G8**
Water to cement ratio	W/C (-)	0,31	0,47	0,59	0,57	0,63	0,9
Compressive strength	Rc_{sat} (MPa)	72,9±1,4	43,3±0,8	43,8±1,5	40,5±0,7	38,3±0,8	20,2±1,0
Static deformation modulus	E_{sat} (GPa)	35,5±0,9	28,4±0,9	27,7±3,1	27,9±0,4	27,4±2,8	21,3±1,1
Mean density of the slab	ρ_{sat} (kg/m^3)	2441±8	2469±11	2457±13	2447±7	2455±12	2405±11
Porosity measured by water saturation	ϕ (%)	12,5±0,3	14,3±0,2	15,5±0,5	16,0±0,7	15,9±0,8	18,1±1,0

3.2 Analysis of the inverted deformation moduli

In the following paragraph, the Poisson ratio and the dynamic Young modulus were obtained by inverting the frequency triplet ($f4$, $f5$, $f6$) in all saturation conditions and degradations studied. The Poisson ratio obtained is varying from 0.17 to 0.24, classical values for concrete, but with important dispersions for a same concrete according to the water content. As it can not be validated by other measurements, it will not be analyzed further.

As expected, E_{dyn} is systematically greater than E_{stat}, both in dry and saturated state (Figure 2.a). Both static and dynamic moduli show a very good correlation between all concrete mixes studied with widely distributed porosities. E_{dyn} is also very well correlated with porosity, one of the main durability indicators (Figure 2.b).

The most interesting inference concerns the evolution of the bulk dynamic modulus E_{dyn} against the water content of the concretes (Figure 3.a). When the water content W decreases from the saturated to the dry state, the E_{dyn}-modulus decreases to reach a minimum at low water content and then increases to its highest value. This phenomenon is corroborated by the surface wave velocity measurements in the same project [7]. The celerity variations could be explained by a competition between the density and the modulus rise, at low water content, the capillary forces increase the density whereas the increase in modulus is dominating at high water content [8,9]. The modulus evolution follows the celerity variations. It was observed in sandstone and limestone [8,9] but not in cementitious materials [10,11], because the saturation degree corresponding to the minimum had not been reached nor studied.

Concerning the carbonation, the impact-echo measurements seem not very sensitive to this degradation phenomenon (Figure 3.b) because the result correspond to the mean value on the whole thickness carbonated or not. Only concrete G8 was completely carbonated (6cm from both faces) but it is not conclusive because the slight decrease is of the same order of magnitude as the result dispersion due to the material dispersion.

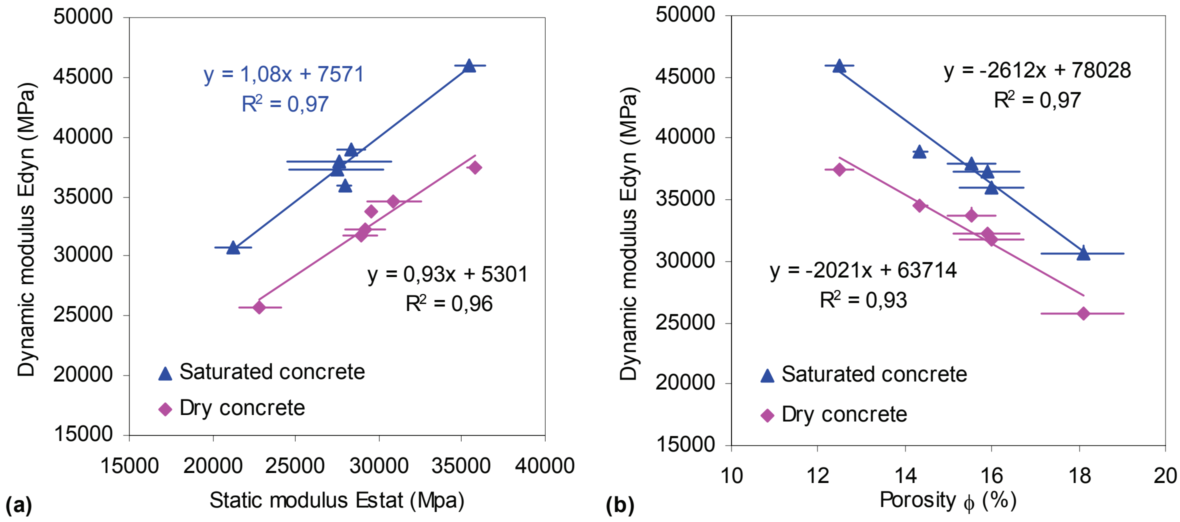

Figure 2. *a. Dynamic modulus compared to the static one in dry and saturated concrete b. Dynamic Young modulus versus the porosity*

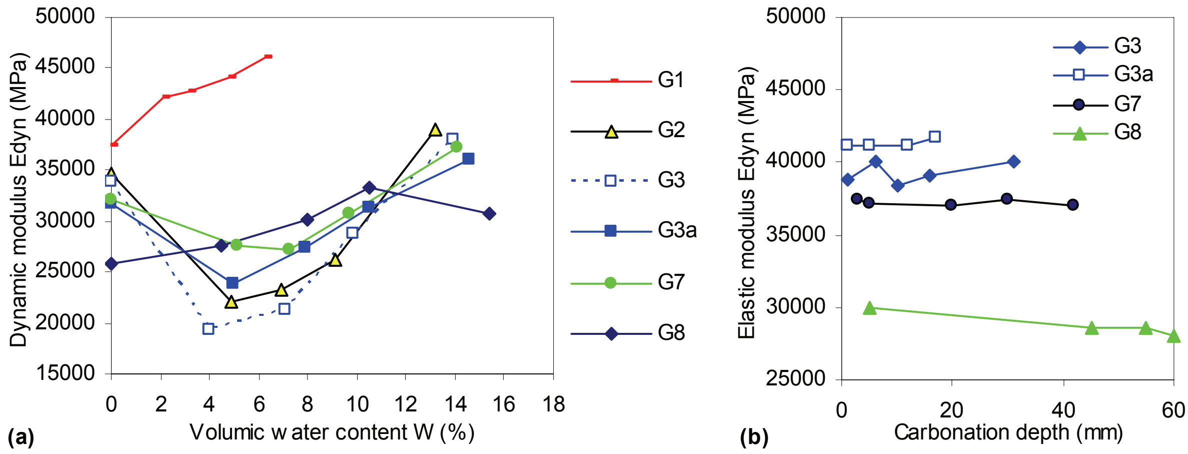

Figure 3. *a. Dynamic Young modulus versus the volumetric water content b. Effect of carbonation on the dynamic modulus*

4 Conclusions

The thin slab and beam theories make it possible to better understand the behavior of the reduced size slabs submitted to impact-echo tests. It is then possible to identify several frequencies corresponding to resonance or stationary modes, to obtain their analytical expression and then to proceed to an inversion in order to calculated the Poisson ratio ν and the dynamic Young modulus E_{dyn}. The experimental results for 6 different concretes in different conditions show that: E_{dyn} is greater than E_{stat} and both very well correlated; E_{dyn} is a linear function of the porosity for the studied concrete; the mechanical modulus is sensitive to the water content. As a consequence, it is necessary to use simultaneously several ND methods to distinguish the porosity and the water content effect [12]. The effect of the carbonation on the bulk dynamic modulus is not yet clear because it is very difficult to carbonate completely 12cm-thick slabs of not very porous concrete.

Acknowledgements

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